Charge and Mass of the Electron

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We use high-energy electrons emitted from an ^{133}Ba source to measure the relationships between velocity, momentum, and energy. We find that relativity predicts these relations much better than classical mechanics, and we measure an electron mass of 534 ± 8 keV and an electron charge of $5.5 \pm 0.1 \times 10^{-10}$ statcoulombs. We consider the possible effects of a non-uniform magnetic field.

I. INTRODUCTION

The success of classical mechanics as developed by Isaac Newton in the 17$^{\text{st}}$ century stood for over 200 years as one of the great triumphs of Enlightenment science. However, at the turn of the 20$^{\text{st}}$ century, it became clear that this theory had certain failings at the extremes of human experience. When objects were moving at very high speeds, Newtonian mechanics gave increasingly inaccurate predictions.

Newtonian mechanics was especially hard to reconcile with the newly developed electromagnetic theory. This seemed required the existence of a luminiferous æther permeating the universe through which light could pass, but which a series of increasingly complex experiments failed to detect. At this time, Albert Einstein built upon insights by various contemporaries to develop the theory of special relativity, which posited that light had a constant velocity regardless of reference frame, and derived many surprising facts that have been confirmed to spectacular accuracy by theory[4].

II. THEORY

Newtonian mechanics predicts a linear relationship between velocity and momentum, and a quadratic relationship between energy and momentum:

$$p = mv; K = p^2/2m \qquad (1)$$

On the other hand, relativistic dynamics adds corrections which become relevant at high velocities:

$$\gamma = (1 - (\frac{v}{c})^2)^{-1/2} \qquad (2)$$

$$p = \gamma mv \qquad (3)$$

$$K = mc^2(\gamma - 1) \qquad (4)$$

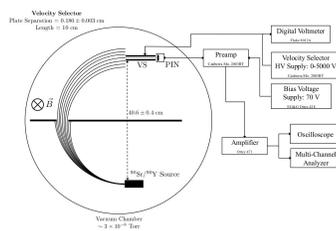* [pranjal@mit.edu, jfeather@mit.edu](mailto:pranjal@mit.edu)

FIG. 1. High-energy electrons ejected from the radioactive source curve through the cyclotron magnetic field and travel through a velocity selector to hit a PIN diode.

In the small v limit, these approach the Newtonian equations. In the large v limit, $K \to pc$ and p grows asymptotically as $v \to c$.

III. EQUIPMENT

To determine the charge and mass of the electron, high-energy electrons are ejected from a ^{133}Ba source and travel along a circular path due to a magnetic field generated by a magnetic coil. They then travel through a velocity selector and hit a PIN diode. These three instruments allow for measurements of the momentum, the velocity, and the energy of the electron.

III.1. Magnetic Field

To generate a uniform magnetic field over the path of the electrons, a high-power DC current source delivers up to 5.75 A at approximately 150 V through a thick copper wire that is wound into loops around the latitudinal lines of a sphere. The portion of the sphere through which the electrons travel is kept at a vacuum of approximately 3×10^{-5} Torr, minimizing the scattering of the electrons by air molecules.

Manipulating the current on the coils sets the magnetic field, which determines the momentum of the par-

ticles that reach the velocity selector. The centripetal force on the electrons is given by $F = \frac{Bve}{c}$. Since $F = \frac{dp}{dt} = \omega p = \frac{vp}{r}$, where r is the radius of the electron beam $(20.3 \pm 0.2\,\mathrm{cm})$, we get the relation $p = \frac{erB}{c}$.

III.2. Velocity Selector

When in the presence of perpendicular electric and magnetic fields, charged particles moving perpendicular to both will be deflected depending on their velocity. The net force is zero when the forces from the electric and magnetic fields cancel; so $Bve/c = Ee$ and $v = cE/B$. Given the velocity and momentum, and the relation $p = \gamma mv$, the relativistic prediction for the E field needed to cancel out a B field is $E = \frac{B^2 k}{\sqrt{1+B^2 k^2}}$, where $k = er/mc^2$. On the other hand, the classical relationship is much simpler. We have $p = mv$, so $E = kB^2$.

The fit parameter gives us the charge-mass ratio of the electron. Since electric and magnetic fields both impart a force proportional to the charge of the particle, manipulating electric and magnetic fields can only find the charge-mass ratio of a particle, not either quantity independently.

III.3. Radioactive Source and PIN Diode

However, knowing the particle's energy allows determination of its mass, based on equation (4).

The only remaining obstacle is the determination of K. For this, we use a PIN photodiode, which is a semiconductor device with an undoped layer between a P layer and an N layer. The device is reverse-biased at approximately $50\,\mathrm{V}$, so high-energy electrons and photons hit the undoped region and free a large number of electrons, which create a current due to the biasing field.

This process requires calibration to determine the relation between the PIN output voltage and the particle energy. For this purpose, we use a ^{133}Ba source placed next to the PIN photodiode for calibration purposes. This decays to ^{133}Cs through electron capture, emitting several gamma rays and x-rays. These present in the PIN output as Compton edges and photopeaks with known energies[2].

We use a ^{90}Sr source to generate the high-energy electrons. This decays to ^{90}Y, emitting an electron with maximum energy $0.546\,\mathrm{MeV}$. This in turn decays to ^{90}Zr, emitting an electron with energy up to $2.27\,\mathrm{MeV}$.

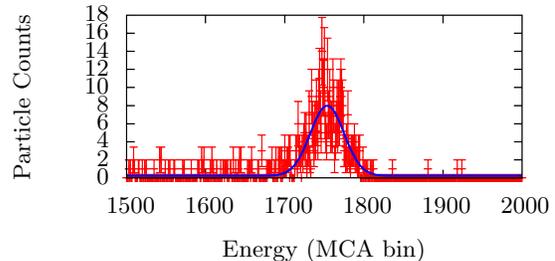Bin counts for selector voltage = 4.3 kV at $B = \sim 100$ gauss

FIG. 2. The output of the MCA for a single (B, E) pair is fit to a gaussian to determine the number of electrons admitted by the voltage selector

IV. PROCEDURE

The PIN was calibrated as above, using the ^{133}Ba source. The magnetic field was calibrated by putting a hall probe at various points in the magnetic field, then sweeping through a set of coil currents to determine the magnetic field the electrons traveled through. Ultimately, the magnetic field ended up as a fit parameter, and only the current-field linearity was tested with the calibration.

To take the measurements, ten selector voltages over a range of about $1\,\mathrm{kV}$ were used for each of ten coil currents from 3 to 5.25 A (corresponding to 60 to 105 gauss). For each (E, B) pair, a gaussian cuve was fit to the signal observed on the MCA, as in Figure 2. The area under the gaussian was extracted from each (B, E) pair, and for each B field, these areas were fit to a gaussian. The fits for each B field are given in 3. The center of the fit, E_{opt} was then the E field that canceled that B field, and these were used to determine the properties of the electron.

V. RESULTS

V.1. PIN Diode Calibration

The spectrum observed by the PIN diode with the calibration source is given in Figure 4. A highly linear relation $(\chi^2 = 0.63)$ was observed between the measured photopeaks and their known energies[2]. Because of the quality of this fit, the Compton edges were not taken into account.
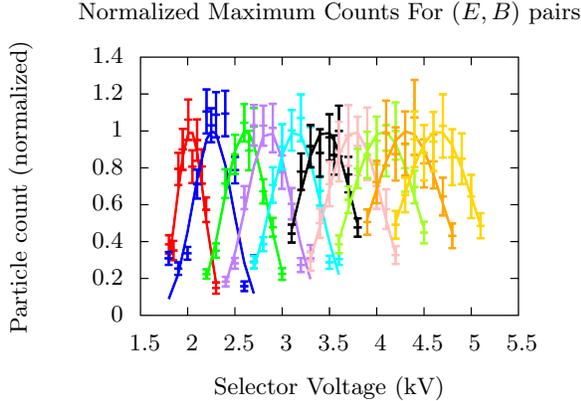
Normalized Maximum Counts For (E, B) pairs

FIG. 3. For a given B field, the count rates for each E field are normalized and fit to a gaussian. The center of this gaussian is the E field that cancels out that B field, so $\beta = \frac{E}{B}$

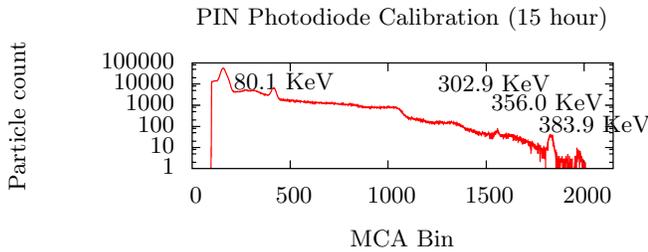PIN Photodiode Calibration (15 hour)

FIG. 4. The ^{133}Ba source produces distinctive photopeaks and Compton scattering lines.

V.2. Magnetic Field Calibration

Measuring precisely the magnetic field in the beam path of the electron proved extremely difficult. Placing the hall effect sensor in different locations throughout the sphere yielded magnetic field measurements that varied over 10%. However, the magnetic field at any given point was extremely linear, and although the magnetic field might vary spatially along the beam path, the total curvature of the electron comes from the integral of the magnetic field, which remained constant. Thus, the coil current-magnetic field ratio could be used as one of the fit parameters for the relativistic velocity-momentum equation.

We found an average magnetic field value from the hall probe measurements of $19.78 \pm 3.15 \, \text{G A}^{-1}$. From the relativistic fit, we found a value of

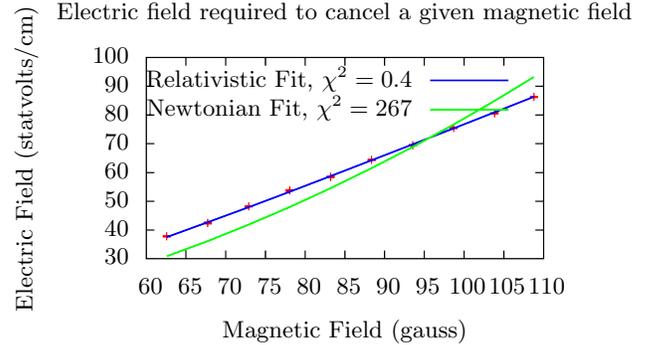Electric field required to cancel a given magnetic field

FIG. 5. The relativistic velocity-momentum relation fits the data significantly better than the classical relation. The fit determines $e/mc^2 = 6.4 \pm 0.1 \times 10^{-4}$ statcoulombs per erg

$20.22 \pm 0.07 \, \text{G A}^{-1}$. Our measured values for the electron charge and mass were particularly sensitive to this ratio, so even this two percent change significantly affected our results.

V.3. Charge-Mass Ratio

To test the classical and relativistic momentum-velocity relations, the (B, E_{opt}) pairs were plotted and fit to the momentum equations (4) and (1). As seen in figure 5, the relativistic relation fits the data well ($\chi^2 = 0.35$), while the classical relation does not fit the data at all ($\chi^2 = 266$). We found from the relativistic fit a charge-mass ratio of $6.4 \pm 0.1 \times 10^{-4}$ statcoulombs per erg. While the experimental error on this is quite low, this value differs significantly from the known value of 5.9×10^{-4} statcoulombs per erg.

V.4. Kinetic Energy and Mass

Next, we tested the relationship between velocity and energy to measure the mass of the electron. We first attempted a classical fit, shown in Figure 7. This was clearly a poor fit, with a χ^2 of 297, confirming that classical mechanics fail at high velocities. The relativistic fit, however, was quite good. We obtained a χ^2 of 0.4, indicating a good fit, and obtained a value of $m_e c^2 = 534 \pm 6 \, \text{keV}$. This is an overestimate of the known value of $511 \, \text{keV}$.

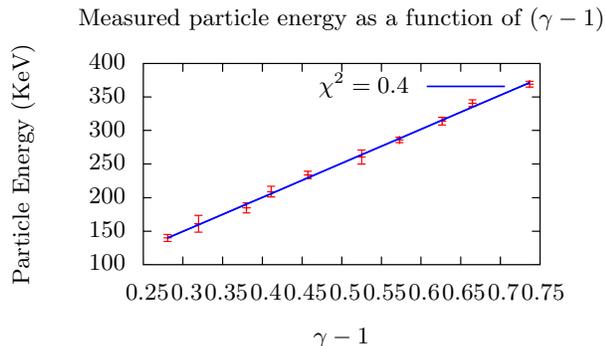From the electron mass and charge-mass ratios, we obtain a measurement of $e = 5.5 \pm 0.1 \times 10^{-10}$ stat-

Measured particle energy as a function of $(\gamma - 1)$

$\chi^2 = 0.4$

FIG. 6. The energy is a linear function of $1 - \gamma$. This fit determines $m_e c^2 = 534\,\text{keV}$.

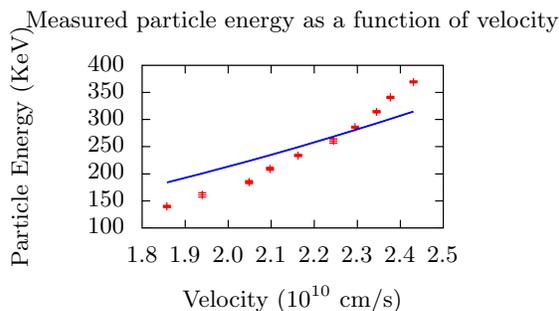Measured particle energy as a function of velocity

FIG. 7. The classical fit for $K = \frac{1}{2}mv^2$ works very poorly ($\chi^2 = 297$)

coulombs, which is again an overestimate of the known value of 4.8 statcoulombs.

VI. ERRORS

The fact that both the mass and charge-mass ratios of the electron were several σ away from the known values combined with the low χ^2 values for the fits suggests the presence of some unaccounted for statistical error.

Earlier, we stated that the nonuniformity in the magnetic field was irrelevant to the results, but it is possible that this is not the case. We rely on the magnetic field in the voltage selector being equal to the magnetic field in the entire sphere, which is probably not the case, and is likely the cause of our systematic error. In fact, increasing the selector field by 1% and decreasing the shell field by 1% produces values much closer to the known values.

We considered the possibility of coil heating causing a drift in the magnetic field over time, but we did not notice significant heating over time with a cooling fan pointed at the sphere.

In addition, our χ^2 values tended to be quite low - for both the velocity-momentum and the gamma-energy relation, they were around 0.4. This indicates that there are correlations in the error that we have not taken into account. These probably come from the errors in the dimensions of the apparatus that we considered as uncorrelated but are actually the same in all the trials.

VII. CONCLUSION

We found strong evidence of the validity of special relativity over at high velocities. We measured the electron charge and mass with relatively high precision, but low accuracy. Our calculated $m_e c^2$ of 534 ± 8 keV is not quite in agreement with the known value of 511 keV, and our calculated e/mc^2 of 6.4 ± 0.1 statcoulombs/erg is again somewhat higher than the known value of 5.8 statcoulombs/erg. We suspect that these deviations are due to a non-uniformity in the magnetic field.

To compensate for this, it would be useful to take a more precise measurement of the magnetic field at the precise points which the electron beam travels through. Since the experiment is sensitive to small changes in the magnetic field, it would be useful to use a 3-axis magnetometer that is less sensitive to alignment issues.

[1] Philip R. Bevington and D. Keith Robinson. *Data Reduction and Error Analysis for the Physical Sciences.* McGraw-Hill Higher Education, 2003.

[2] Richard B. Firestone. Table of radioactive isotopes, 2013.

[3] MIT Department of Physics. Relativistic dynamics: The relations among energy, momentum, and velocity of electrons and the measurement of e/m. February 2012.

[4] Wikipedia. History of special relativity, 2012. [Online; accessed 5-December-2012].